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Remedial Instruction for Teaching Multiplication to Children with Dyscalculia in Inclusive Primary School

An Experimental Study

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Abstract

The article explains the difficulties encountered by children with dyscalculia — a specific learning disability. The authors deal with one of the fundamental arithmetical operations, i.e. multiplication. The study aims at developing remedial arithmetical programme to teach children with dyscalculia. The effectiveness of the programme has been studied through experimental design. The authors have identified common errors in multiplications and compared them in children with and without dyscalculia. The study has implications for teaching multiplication in inclusive schools.

Key words : Remedial Instruction, Dyscalculia, Inclusive Education.

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ARITHMETIC represents an important area of instruction in schools — proficiency in arithmetic is expected beyond the classroom. Some children, though they have normal intelligence, fail to achieve this basic skill at an expected rate. This may be due to specific learning disability in the area of arithmetic. Learning disability refers to a specific retardation or disorder in one or more of the processes of speech, language, perception, reading, spelling or arithmetic (Kirk, 1968). This does not include learning problems which are due to sensory handicap, motor problem, mental retardation, emotional disturbance or adverse environmental factors. The term dyscalculia is used to represent this specific learning disability(LD).

Dyscalculia is most commonly observed among learning disabled children. It could be a manifestation of disabilities in other primary academic areas such as language, reading and writing. It can also exist independently in the absence of other disabilities.

Characteristics of Children with Dyscalculia

Children with arithmetic disability usually have normal or above average intelligence. That is why they are not differentiated from normal population. Only a closer look at them reveals that they differ from their age-mates.The knowledge of their characteristics and difficulties usually experienced by them are useful in indentifying are listed below :

1. They have poor visual memory, visual recognition and sequential memory. They show difficulty in reading and writing numerals and have difficulty in recognising categorical structure of numbers. Sequence of steps to be used in various arithmetic operation cannot be followed by them.
2. Many students with arithmetic disability are deficient in visual spatial organisation and non-verbal integration. So, the children may have problem in acquiring the pre-requisite skills necessary for arithmetic like distinguishing differences between shapes, sizes and quantities.
3. They have disturbance in visual motor integration, thus cannot write that they intend to write.
4. They do not have a strong sense of direction, have problem in understanding concept of time, money and measurement.
5. They have poor visual discrimination and difficulty in differentiating shapes and symbols.
6. They are unable to choose the principles for solving problem as they have poor reasoning ability. Even if, they can do fundamental operations they do not know when to use the particular operation depending upon the demand of the situation.

7. Sometimes they show inconsistency in doing arithmetic operations as they are not clear with either the procedure or the operation.

The number of students with difficulty in arithmetic is quite large. They may experience the above difficulties and exhibit the characteristics in different combination and in varying degrees.

Importance of the Study

In the present article, the authors deal with only one of the fundamental operations—multiplication, though all the fundamental operations were considered in the original study.

Multiplication process is a complex task and the concept is confusing. It is assumed that normal population in classroom develop their skill for this complex process automatically. But an alarming 6% of our population, who suffer from Dyscalculia, find it very difficult to understand the concept of multiplication. If it is not remediated at the right time, it leads to problems in other areas of arithmetic where multiplication becomes a prerequisite. Here comes the need to develop a remedial instruction programme for such children so that their performance will match to that of their peers.

Many earlier investigators observed specific difficulties in multiplication among dyscalculic children. Some of the important observations are given below.

Ramaa (1990) in her investigation report observed that dyscalculics exhibit homogeneity in terms of arithmetic difficulties experienced and types of errors committed by them. She observed that dyscalculics multiply in a wrong sequence.

Confusion in symbol was noticed by Wagner (1980) among LD children. McLeod and Armstrong (1982) listed multiplication of whole numbers as one of the areas of difficulty for children with learning disability.

Children with dyscalculia were found to be failing to recall tables and carry numbers in multiplication by case studies conducted by Cohn (1968, 1971).

In a recent study by Ramaa and Gowramma (1999) an attempt was made to analyse the arithmetic errors committed by three Class VIII students with LD. These students were undergoing remedial instruction in a special centre for one year. These students were administered the Arithmetic Diagnostic Test (Ramaa, 1994) before starting remediation. It was surprising to note that all of them exhibited basic errors while doing multiplication sums.

This shows that the arithmetic difficulties may become persistent even at the higher grades of schooling unless and otherwise their difficulties are corrected at the early stage. This emphasises the need for continuous remedial instruction for CWD throughout their schooling.

Though there are studies of the above sort to explore and analyse the

types of difficulties faced by Children with Dyscalculia (CWD), the attempt to remediate these difficulties are very much limited. In a study by Lombardo and Drabman (1985) the investigators attempted two procedures to teach multiplication to LD children and compared the results. The experimental groups were suggested to use verbal mediation while doing multiplication whereas the other group did it without overt verbalisation of the procedure. The performance of the experimental group was significantly better than that of the control group. This shows that with simple principles and strategies, it is possible to make CWD to learn multiplication.

The reasons for limited attempts to develop remedial instruction materials for dyscalculics may be due to some of the new trends in mathematics education. Fischbein (1990) in his article *Psychology and Mathematics* has reviewed some recent trends that tend to shape the future course of mathematics education. He has observed that from the mid seventies the researchers had devoted much time and effort to the problems and concepts of 'artificial intelligence'. Those researchers try to translate the problems of mathematics education in terms of the information processing approach. In the place of developmental and cognitive psychology as major sources of influence on mathematics education, modern perspective of cognitive science has taken upper hand. As per this perspective, short term and long term memory, working memory and routines and sub-routines are considered more important. No doubt, artificial intelligence metaphors and computer assisted instruction have beneficial effects both theoretically and didactically. Through computer assisted instruction, students may improve their capacity to analyse their own reasoning processes, may become more active or may be involved actively and interact intelligently with the computer in defining and solving the original problems.

But certainly instructional process through computer has its own limitations. Seriously, computerised instruction may become a real danger especially when its use is emphasised as an integral part of school education. Because in the opinion of Dreyfus and Dreyfus (1986) expertise cannot be reduced to an aggregate of elementary skills acquired stepwise at lower levels. Fischbein (1990) particularly insist on this point because he felt that there is a tendency to include the computer in teaching programmes without a careful consideration of its psychological and didactical effects. Dreyfus and Dreyfus (1986), and Fischbein (1990) agree that the experts react globally, by intuition, on the basis of rich experience. Formerly learnt rules do not play an important role any more at expert level. However, this intuition is not irrational but scientifically analysable (Fischbein, 1987).

This suggests that computer cannot always take the task of

simulating every type and every level of our intelligent behaviour and also computer cannot tutor every aspect of a learning process. So, there is a need for developing remedial instructional methods and materials which can take care of all the abilities required for mathematics learning — concepts, skills and problem solving strategies. This type of materials and methods will be more effective when the activities involved has rich experiences involving real life situations, social interaction with the peers and adults like teacher and parents which is especially true with children with LD. The present study is an attempt to develop a remedial instruction material for teaching multiplication to CWD, keeping in mind the above assumptions.

Objectives

The specific objectives of the study were :

1. To identify children with dyscalculia in primary schools.
2. To analyse the errors committed by dyscalculics and normal children of Classes III and IV while doing multiplication sums.
3. To develop a remedial arithmetic programme to teach multiplication to CWD.
4. To find out the effectiveness of the remedial arithmetic programme developed to teach multiplication.

Hypothesis

The present study attempted to verify different hypothesis. The one which is most relevant to the present part of the study is given below.

The remedial arithmetic programme developed in the study will be effective in improving the performance of dyscalculic children in multiplication.

Methodology

Fifteen primary schools were selected in Mysore city depending upon the feasibility to administer various tests and also to carry out remedial programme. A standard procedure for identifying children in Classes III and IV with Dyscalculia was used based on exclusionary criteria. Children who exhibited arithmetic retardation of atleast two years, compared to their grade level were considered for the study. These children had normal sensory, visual and auditory functioning, no serious social and emotional problems and had adequate intelligence. Out of the total population of 1400, using the exclusionary and inclusionary criteria using various tools and techniques, 82 children were identified as CWD.

Selection of Normal Children

To compare the performance of CWD with normal children, a matched group of normal children were selected by obtaining a list of students who perform at an average level in Classes III and IV by the class teacher. The teacher's opinion was cross-checked through informal assessment.

On the basis of the responses to items of arithmetic diagnostic test, a

detailed analysis of the errors was made, in doing all the sums. In the present article, only errors related to multiplication is discussed. The analysis was done with two purposes, the first purpose was to identify the types of errors and to understand the probable reasons for the errors/ difficulty/deficiency. The second purpose was to compare the types of errors committed by both the groups in terms of percentage. Tables 1 and 2 give the results of the analysis.

TABLE 1

Common Errors Noticed in Multiplication in Normal Children and CWD

S. No	*Errors*	*Example*	*Probable Reasons*
1.	Does not attempt	5=x6	1. Does not know the multiplication fact 2. Cannot verbalise the numerical expression
2.	Adds the number	35 +3 ---- 38 ----	1. Confusion of symbols 2. Does not know the multiplication fact 3. Transfers the procedure learnt earlier, which is not appropriate.
3.	Cannot carry digits	15 3	1. Lack of procedural knowledge of multiplication 2. No place value concept.
4.	Cannot multiply 'O'	50 x2 10 ---	No '0' concept
5.	Does not know multiplication facts correctly	33	1. Poor memory 2. Lack of attention
6.	Forgets to add the digit carried	15	1. Lack memory 2. Lack of attention
7.	Adds the carried digit before	15	1. Confusion of procedure multiplying

Comparison of Errors among CWD and Normal Children

In order to compare the errors committed by both normal children (N=30) and CWD (N=82), Arithmetic Diagnostic Test

TABLE 2

Percentage of Children who Exhibited Different Types of Errors/Deficiency in Multiplication

S. No.	*Error/Deficiency*	*Percentage in Group A**	*Percentage in Group B***
1.	Lack of concept and skill of multiplication	60	0
2.	Added the number instead of multiplying	37	6.89
3.	Could not perform multiplication involving more than one digit	100	44.80
4.	Did not know multiplication when 'o' was present	100	41.00

* Group A - Children with Dyscalculia
** Group B - Normal children

From Table 2, it is observed that none of the dyscalculics knew the multiplication of more than one digit and when 'o' was one of the factors. Around 40 per cent of normal children also had problem in the same area. It is thus evident that normal children also have committed errors and exhibited deficiency in multiplication. However, compared to dyscalculics their percentage is low. Earlier studies by other investigators also support difficulties experienced by CWD in all the arithmetic operations in general and multiplication in particular.

Rourke (1982), Strang and Rourke (1985), Wood (1980), Wagner (1981) noticed the error in visual detail, which leads to misreading of computational signs (+ and x). Kosc (1974) noticed that dyscalculics are generally poor in acquiring the concept and performance in mathematical operation. These types of difficulties were also observed by Luria (1966a, 1966b, 1970). Difficulty in multiplication of whole numbers was observed by McLeord and Armstrong (1982), Ramaa (1990), Cohn (1968, 1971).

Ramaa and Gowramma (1999) observed the above types of difficulties in doing multiplication among LD children of Class VIII.

Planning and Preparing the Remedial Instruction Material

Based on the errors/deficiencies exhibited by children, also keeping in mind the general principles suggested by various researchers a remedial programme to teach multiplication was developed.

Some of the general principles to teach arithmetic :

1. Readiness skills should be emphasised.

2. Teach same concept in different representations.
3. Use simple vocabulary.
4. Teaching should have direct impact on child's perceptual faculty.
5. Foster cognitive development and arithmetic skill simultaneously.
6. Individualise the instruction.
7. To compensate the areas of deficit, multisensory attack should be made.
8. Associate arithmetic to daily life.
9. Arrange instruction in learning hierarchy.
10. Immediate and continuous success for the students is basic to math instruction.

Specific principles to teach multiplication

1. To compensate for short-term memory performance cues should be used to remember steps while doing multiplication.
2. The terms and symbol of multiplication should be used frequently for better retention.
3. Basic multiplication facts should be taught through different methods.
4. Multiplication should be taught as an application to daily life.
5. Activities should be drawn from situations dear to children.

Main Features of the Remedial Programme

Keeping the above principles in mind, the remedial programme to teach multiplication to children with dyscalculia was developed. Some of the main features of the programme are :

1. The programme was developed in such a way that it caters to the need of majority of children who have problem in learning multiplication.
2. Each lesson has a specific objective.
3. Activities are arranged in sequential order.
4. Achieving the objective of the previous lesson is a prerequisite to go to the next lesson.
5. Lessons are made very short requiring 15-20 mts covering a specific concept.
6. Each activity emphasises on verbalisation and visualisation for concept clarity and acquisition of skill.
7. Learning activities were linked to the previous activities.
8. Concepts were taught using concrete materials. Slowly, it was shifted to semi-concrete and finally the abstract form of numbers were used.

Here, since we are discussing only the remedial programme developed for teaching multiplication to children with dyscalculia, a list of prerequisite concepts is given here. A programme was developed to cover them also, before coming to multiplication.

Prerequisite Skills to Learn Multiplication

1. Number Concept
2. Place Value
3. Addition

To make sure that children are ready to learn multiplication, an evaluation/review of the prerequisite concepts is necessary.

The programme developed was activity-oriented. The remedial instruction programme of multiplication to teach children with dyscalculia has 23 lessons and 15 sessions were required to complete the administration of the programme to a selected sample of dyscalculics. The following are the specific objectives of each lesson in sequence.

Lesson 1

To learn the terms and symbol of multiplication.

Lesson 2

To read the numerical expression when 'x' symbol is used in different ways.

Lesson 3

To understand the concept of multiplication (without using numerical expression).

Lesson 4

To know that multiplication is nothing but repeated addition.

Lesson 5

To learn multiplication tables 0-5.

Lesson 6

To understand the procedure of multiplication when 0-5 numbers are used.

Lesson 7

To solve daily life problems involving the operation multiplication, where numbers from 0-5 were involved.

Lesson 8

To learn multiplication tables 6-10.

Lesson 9

To understand the procedure of multiplication when numbers from 0-10 were used.

Lesson 10

To solve daily life problems involving the operation of multiplication where numbers from 0-10 were used.

Lesson 11

To learn the skill of multiplying a 2-digit number by a 1 digit number.

Lesson 12

Multiplying a 2-digit number by 10.

Lesson 13

Multiplying a 2-digit number by multiples of 10.

Lesson 14

Multiplying a 3-digit number by 1 digit number.

Lesson 15

Multiplying a 3-digit number by 10.

Lesson 16

Multiplying a 3-digit number by multiples of 10.

Lesson 17

Multiplying a 2-digit number by a 2 digit number.

Lesson 18

Multiplying a 3-digit number by a 2 digit number.

Lesson 19

Multiplying a 2-digit number by 100.

Lesson 20

Multiplying a 2-digit number by multiples of 100.

Lesson 21

Multiplying a 3-digit number by 100.

Lesson 22

Multiplying a 3-digit number by multiples of 100.

Lesson 23

To solve simple daily life problems involving multiplication.

Sample Lesson

The planning phase of the remedial procedure consisted of writing out the remedial programme on the above mentioned lines. One sample lesson is given below.

Lesson 14

General Objective : To enable the children to understand the concept and procedure of multiplication.

Specific Objective : Multiplying a three digit number by a single digit number.

Example 1

Multiply 369 by 2

```
 369
 x 2
```

I Method : Multiply each of the number by two and add them.

```
9x2 =    18
60x2 =  120
300x2 = 600
         738
```

II Method : Multiply nine by two. Write units in unit's place and carry the tens to tens place.

```
  1
 369
 x 2
   8
```

Multiply six by two, add the carried digit, write the tenth digit in tens place, carry the hundred's digit to hundred's place.

```
  1
 369
 x 2
 38
```

Multiply three by two, add the carried digit, write the answer in hundred's place.

```
 369
 x 2
 738
```

The students imitate the same problem once with the model and next without the model. Then, as many as 30 similar sums for exercise were given.

Validation of the Remedial Instruction Material of Multiplication

In order to validate the effectiveness of the remedial programme developed, a pre-test - post-test single group design was considered to be appropriate because of the inter-individual differences among identified children and non-availability of the matched control group.

The treatment was given to eight children out of the 82 children identified in the initial phase of the study. However, care was taken to see that these eight children were the representatives of other children with dyscalculia in terms of errors committed in all the fundamental operations.

The programme was administered daily for an hour in 15 sessions on working days in schools. Continuous evaluation was done and reduction in error was noticed. Post-test scores indicated that on the whole the programme was effective.

Analysis of the Experimental Data

In order to verify the effectiveness of the programme in developing multiplication skills among CWD the scores obtained during pre-test and post-test were subjected to quantitative analysis by using 't' test of significance. The table below gives the mean, SD and 't' ratio.

TABLE 3

Mean Performance on Pre-test and Post-test in Multiplication

Tests	*N*	*M*	*SD*	*t-ratio*
Pre-test	8	0.75	1.48	
Post-test	8	29.88	2.85	25.61*

* t-value significant at 0.01 level.

It can be understood from the table that the programme developed is effective in improving multiplication skills among CWD.

In addtion to comparing the mean performance at pre-test - post-test level on the multiplication scores, further analysis was done with reference to different criterion measures relating to multiplication.

TABLE 4

Percentage of Cases Passed in Pre-test and Post-test in Multiplication — One Digit Number x One Digit Number

Item	*Percentage in Pre-test*	*Percentage in Post-test*
1.	25	100
2.	12.5	100
3.	0	100

TABLE 5

Percentage of Cases Passed in Pre-test and Post-test in Multiplication — More than One Digit Number × One Digit Number

Item	*Percentage in Pre-test*	*Percentage in Post-test*
1.	0	100
2.	0	100
3.	0	87.5
4.	0	100

TABLE 6

Percentage of Cases Passed in Pre-test and Post-test in Multiplication — More than One Digit Number × One Digit Number

Item	*Percentage in Pre-test*	*Percentage in Post-test*
1.	0	100
2.	0	100
3.	0	100
4.	0	100

TABLE 7

Percentage of Cases Passed in Pre-test and Post-test in Multiplication — Multiplying and Arranging the Product in Ascending Order

Item	*Percentage in Pre-test*	*Percentage in Post-test*
1.	0	100

TABLE 8

Percentage of Cases Passed in Pre-test and Post-test in Multiplication — Problem Solving

Item	*Percentage in Pre-test*	*Percentage in Post-test*
1.	0	100
2.	0	100

These Tables reveal that there was a significant improvement in the performance of all the subjects of the experimental group at the post-test stage in the area of multiplication.

Most of the subjects did not have the concept, procedure and skill of multiplication during pre-test. All the subjects performed at the level expected of grade IV children after remediation as revealed in the post-test.

In some of the earlier studies also (Rozario and Kapur, 1992; Pandit, 1993; Kosc, 1977 and Sharma, 1993) a significant improvement was observed in multiplication among LD children after remedial instruction.

Major Findings

1. The research findings show an alarming 6% incidence of dyscalculia in primary school.
2. A systematic procedure to identify children with dyscalculia in primary school was developed.
3. Error analysis related to multiplication skills helps in understanding the difficulties experienced by children with dyscalculia.
4. Major shortcoming in the performance of multiplication was noticed among dyscalculic children. Multiplication of zero and multiplication of numbers when the digits are more than one were observed to be the most difficult areas.
5. A suitable remedial instruction programme to teach multiplication to children with dyscalculia was developed.
6. All the children in the experimental group showed better performance after remediation in multiplication.
7. A remedial arithmetic programme based on the suggested principles, strategies and approaches significantly improves CWD in multiplication.

Implication of Findings for Teaching Multiplication in Inclusive Primary Schools

1. Since the programme developed to teach multiplication to dyscalculia caters to the need of any child, it can be used on anybody who is having problem to learn multiplication due to various other reasons in primary schools.
2. If the specific deficits are identified and faulty strategies adopted to do the operation is rectified, children who are suffering from dyscalculia can be helped to overcome their problem.
3. Though the programme is developed in Kannada, since arithmetic is not language bound, programme can be used anywhere in India and also abroad, with suitable adaptations.
4. Since they have difficulty in prerequisites of multiplication, provision should be made for

remedial instruction in the inclusive schools. Small group or individual instruction would be better.

5. Multiplication was observed to be difficult for most of the normal children also. So, the remedial strategies suggested here can be made use of to teach in the regular classrooms so that it will be helpful to the normal children.
6. Teachers should be trained in methods to teach dyscalculics and other children with disabilities/ difficulties in learning arithmetic.

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